Schwartz

8.4 Axial gauge: $A_0 = 0$.

Carry standard derivation for eqm with $\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 - A_\mu J_\mu$:

EQM: $\partial_\mu F_{\mu\nu} = J_\nu$

p-space: $(-p^2 g_{\mu\nu} + p_\mu p_\nu) A_\mu = J_\nu$

Since $A_\mu = (0, \vec{A})$, we have a 3x3 matrix eq:

$$(-p^2 + p_i p_j) A_i = J_j$$

$-p^2 = -(p_0^2 - |\vec{p}|^2)$, for $A_0 = 0$, it's $|\vec{p}|^2$, so.

$$[\,|\vec{p}|^2 + p_i p_j\,] A_i = J_j$$

The next page shows det $[\,|\vec{p}|^2 + p_i p_j\,] \neq 0$, thus

it can be inverted.

$$M = \begin{bmatrix} |\vec{p}|^2 + p_1^2 & p_1 p_2 & p_1 p_3 \\ p_2 p_1 & |\vec{p}|^2 + p_2^2 & p_2 p_3 \\ p_3 p_1 & p_3 p_2 & |\vec{p}|^2 + p_3^2 \end{bmatrix}$$

$$\det(M) = \left(|\vec{p}|^2 + p_1^2\right)\left(|\vec{p}|^2 + p_2^2\right)\left(|\vec{p}|^2 + p_3^2\right)$$

$$- \left[(|\vec{p}|^2 + p_1^2)(p_2 p_3)^2 + (|\vec{p}|^2 + p_2^2)(p_1 p_3)^2 + (|\vec{p}|^2 + p_3^2)(p_1 p_2)^2 \right]$$

$$+ 2\, p_1 p_2\, p_1 p_3\, p_2 p_3$$

$$= |\vec{p}|^6 + |\vec{p}|^4 p_3^2 + |\vec{p}|^4 p_2^2 + |\vec{p}|^4 p_1^2$$

$$+ |\vec{p}|^2 p_2^2 p_3^2 + |\vec{p}|^2 p_1^2 p_3^2 + |\vec{p}|^2 p_1^2 p_2^2 \quad \text{cancel}$$

$$+ p_1^2 p_2^2 p_3^2$$

$$- |\vec{p}|^2 p_2^2 p_3^2 - |\vec{p}|^2 p_1^2 p_3^2 - |\vec{p}|^2 p_1^2 p_2^2$$

$$\text{cancel} \quad - p_1^2 p_2^2 p_3^2 - p_1^2 p_2^2 p_3^2 - p_1^2 p_2^2 p_3^2$$

$$+ 2 p_1^2 p_2^2 p_3^2$$

$$= |\vec{p}|^6 + |\vec{p}|^4 p_3^2 + |\vec{p}|^4 p_2^2 + |\vec{p}|^4 p_1^2 \neq 0.$$

$$\Rightarrow \boxed{\vec{J} = M^{-1} \vec{A}}, \quad M^{-1} \text{ is trivial to compute}$$

Davidson Cheng

2-21-2024